A mathematical model for product selection strategies in a recommender system

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**ABSTRACT**

Electronic commerce (EC) has become an important support for business. EC is regarded as an efficient platform to bridge the gap between the suppliers and consumers, but how to make use of a huge amount of transaction data and identify potential customers on the internet remains a challenge for an EC company. In particular, to recommend proper products to customers, the preferences of the targeted customers need to be accurately specified and their preferences should be taken into account. This is not only to show the goodwill of the company, but also to retain the customer relation. This study aims to construct a recommender system by focusing on the on-line decision support module with respect to customers' characteristics and supplier's profits. For effective decision support, a mathematical model is developed so that the right product can be recommended to the right person with the best profit for the company. A numerical example is used to illustrate how this model works when both supplier's and consumers' desires are taken into consideration to achieve an optimal Win–Win Strategy for market expansion.

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1. Introduction

With the developments of World-Wide-Web, Electronic commerce (EC) has been increasingly used as a platform for online customers to perform different activities for their daily needs. Among those activities, on-line shopping has been a very successful application. Instead of the traditional on-spot shopping, EC provides alternative ways to get the information of products for purchasing the desired products with the following properties of transparency (Morgan Stanley Dean Witter, 2000):

1. **Price transparency**: participants can learn about the nearly perfect information on price variation from different regions or the suppliers.
2. **Availability transparency**: customer can get the information on which company has his desired service in hand.
3. **Supplier transparency**: who else makes the product?
4. **Product transparency**: it indicates whether there is an alternative product.
5. **Process transparency**: it implies that online customers can have access not only to product information, product pricing and product availability; but also to order status, product career information (Yang, Pan, Wang, & Xu, 2004).

An EC company with the above five transparencies can be regarded as a matured company. However, when an online customer faces so much exposed information, it is difficult for him/her in making a quick and effective decision (The Economist, 1997). With the competitive market and impatient online customers, the need of a customized decision support system is urgent and essential for an EC company so that by providing more helpful information for the customers, the faster and more satisfactory can the decisions be made; the better the retaining opportunity can the company be achieved with higher profits.

From an EC company's perspective, the main problem is how to provide the customers with appropriate goods from thousands of available products. Many EC suppliers use the recommender systems to help the companies find out the preferences of the customers; therefore the right products can be recommended to the targeted customers (Schafer, Konstan, & Riedl, 1999). A well established recommender system can add values to an EC company in several ways. For instance, at first, by a web browser, the customers can retrieve the information they need easily; second, by suggesting additional products for the customers, the company can enhance the cross-selling; and third, by a well designed recommender system, the customers can purchase desirable goods in a more convenient and satisfactory way so that the customer's loyalty can be sustained.

Schafer et al. have introduced a framework of a recommender system and divided it into three stages of the input profiles, recommending methods, and output results (Montaner, López, & Rosa, 2003; Schafer, Konstan, & Riedl, 2001; Schafer et al., 1999). The customer's personal data and preferences are the inputs to the recommender system. From the input data, the adopted recommending method functions and selects the suitable products as an output to the customer. With this kind of structure, the method that makes use of the customer's data for recommendation in the

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A recommender system is the core of the decision support process for the EC suppliers; and the performance of the adopted recommending method would be the measurement of a recommender system.

There were many researches exploiting the customer’s preferences and help the EC website achieve correct product selection by using the techniques of information filtering (Kohrs & Meriadlo, 2001; Sarwar, Karypis, Konstan, & Riedl, 2000). By filtering out the noise of the information provided by the customers, the techniques aimed to find the preferences of the customers, and then recommend proper products to customers. For the EC suppliers with different properties of the selling goods, they would have specific tactics to set up a recommender system. For instance, if the attributes of the goods for a web site could be defined easily (which usually happens when the goods are concrete such as usual commodities and groceries), the web site may introduce the filtering techniques regarding the properties of goods. On the other hand, when the web site is to provide abstract goods such as multimedia or service, the adopted filtering techniques may be considered to find purchasing behavior of the customers since the goods are no longer easily defined.

Although there are different approaches to retrieve needed information for recommendation, a systematic and comprehensive decision model is still lacking. Therefore, not only the time spent on data retrieval is unbearably long; but also the recommended products may not match the customers’ desires. In particular, without a structural model, trace of recommending procedure becomes difficult and thus to achieve the goal of “the right goods for right person” becomes impossible.

With this concern, we aim to construct a general model that could be comprehensively applied to EC web sites as a decision support module so that an optimal recommendation strategy for both suppliers and customers can be developed.

The paper is organized as follows: in Section 2, the literatures related to the framework of a recommender system are discussed. Then, a mathematical model comprising main modules of the recommender system will be proposed in Section 3 and its properties are investigated with strategy development in Section 4. Furthermore, we propose a procedure for the on-line recommending module with an illustrative example in Section 5. Finally, some concluding remarks are given in Section 6 with the suggestion of further research.

2. Framework of a recommender system

According to the frameworks proposed by Schafer et al. (2001), Schafer et al. (1999) and Montaner et al. (2003), the three modules involved in a recommender system include 1. Input sources of the customers’ profiles, 2. Recommending method as the interface mechanism, 3. Output recommendations, which is shown in Fig. 1. In the follows, we shall briefly review the current developments with respective to these three modules.

2.1. Input sources

In general, the input sources of the customer’s profiles include the customer’s preferences from the specific items, preferred item attributes, ratings, and keywords or even purchase history (Schafer et al., 2001). These sources are usually acquired by the web links when customer has clicked. When the desired products are determined to be purchased, the demographic data of the customer are requested for purchase confirmation. These customers’ records provide valuable information to an EC company for customer relationship management. While the frequency of the database usage increases as internet’s users increase, management of these input profiles should be easy of maintenance and retrieval. The common techniques used to maintain the customer profiles are the History-based model (Montaner et al., 2003) and Vector-space model (Chen & Sycara, 1998; Raghavan & Wong, 1986). A History-based model keeps a list of purchases, the navigation history over the internet or the content of e-mail boxes as a customer profile. In the Vector-space model, items are represented by a vector of features or attributes, usually words or concepts, with an associated value. Since Vector-space model is more efficient for computation when large amount of data are used, it has been widely adopted, and so does in our research.

2.2. Output recommendations

Usually, the output turns out to be a suggestion with the information of item type, quantity and appearance (Schafer et al., 1999). The simplest form of a suggestion is the recommendation of a single item. A single item would increase the chance that the customer will seriously consider the item as a desired one. More commonly developed recommender systems are those which provide a recommendation list in/without order of preference for a customer. Besides, some advertising strategies can also be embedded in the recommendation with the display of bundled items. Therefore, it could help enhance cross-selling and up-selling. By comparison with bundled items and a recommendation list, bundled items may include products that are not exactly as customer’s request since they are generated for the purpose of promotion. In contrast to the bundled items, a recommendation list shows a set of products that satisfies the customers at certain degree.

2.3. Recommending methods

For a recommender system, it is critical to find out the customer’s purchasing behavior in a systematic way so that it can support the decision of product selection. The recommending method is a module which serves this purpose of product selection. There are various approaches and issues concerned with the products selection (Liu & Shih, 2005; Yang et al., 2004). Conventionally, there are techniques used to select products by analyzing consumer’s purchasing behavior such as Mapping (Lee, Kim, & Rhe, 2001), Association rule (Hsu, Chen, & Ling, 2004), and the information filtering technology. Because this is related to the core of present study, in the follows, we would discuss these techniques or methods adopted in the recommending process in more detail.

A Mapping process is based on a database which is constructed off-line with the contents of the records of the relationship between consumers and products (Lee et al., 2001). Then, by mapping a new customer to the database, a product that has been purchased by the same type of historical customers can easily be picked up and recommended for the new customer. The mapping process can be regarded as the basic mechanism for the recommender system. In this mechanism, the major concern would be the effective way to determine the interrelated customers of a group. Therefore, the off-line database is usually constructed with the technique of Clustering Analysis. Clustering Analysis holds the idea of grouping the items with the similar properties into one cluster such that the differences within a cluster are minimized and differences between clusters are maximized (Theodoridis & Koutroumbas, 2003). A wide range of applications have been implemented by clustering techniques, and one of those is used to classify the unknown cus-
An off-line database is constructed and divided into three parts: (1) Customers’ profiles, (2) Products’ properties, (3) Relation matrix (Weight matrix). This off-line database is to maintain the customer’s personal data, the properties of the selling goods as well as the relationship in between.

In this paper, we would focus on the on-line recommending module which mainly includes an analytical model proposed for optimal product selection. Therefore, after the customer’s input of the objects. Only other customers’ opinions on the considered objects are relevant and useful in predicting the target customers. The drawback of CF is the requirement of huge amount of rating data. Therefore, it could not function well when there are few or no ratings.

In Table 1, we summarize the concepts and the differences of these two approaches. By taking the advantages of both approaches into consideration, a hybrid approach could be adopted and this is the main stream of current researches.

### 2.4. Summary

From the brief review of the framework of current recommender systems, we can find both merits and deficiencies of each approach. From the viewpoint of managing an EC site and its recommender system, it is more robust and convenient if an analytical model comprising the three modules can be proposed to facilitate the product selection process. For the websites with different properties of goods to sell, the recommender systems would have different types of input sources, and the corresponding recommending mechanism will be a key module which determines the success of a recommender system. With this concern, developing a recommending module that can achieve the transparent requirements of the decision support process and provide a good solution for recommendation purposes is necessary and is our aim of study.

Therefore, in the next section, we shall propose a recommending module that aims to describe the recommendation process of the recommender system, in which an analytical model for supporting the selection decision is particularly emphasized. That is, we shall focus on how to achieve the output recommendations as well as the strategy development when the input sources (customer’s demographic data, personal information, and preferences) are already known.

### 3. The proposed module for decision support process

Consider the flowchart of a recommender system as shown in Fig. 2. When the customer enters the recommender system, he/she is asked to provide the basic attributes such as personal data, preferred item attributes shown on the homepage. Then the data is mapped to an off-line database to match the existing customer’s profile such as the preference weights on products of the similar customer group. When we retrieve the customer’s profile, it is used as the given information to the on-line analytical model for selecting suitable products and yields a recommending list as an output to the consumer.
profiles are retrieved from the off-line database, this model will provide an optimal set of products, that is, a recommendation list to the customer.

In the following section, the on-line recommending module will be discussed in details.

3.1. On-line recommending module

Since the on-line recommending module mainly includes an analytical model, in this study, we shall focus on the development of this model which is to support an EC company so that by considering the customers’ possible budget and preference structure, proper commodities will be recommended to maximize the company’s profit. Let’s first define some notations before we introduce the model with its properties.

3.1.1. Notations

We shall define the notations used, respectively, for products, customers and their relations as below:

3.1.1.1. Products’ properties. Let D be the total number of products, $p_{di}$, in the database, where $d = 1, \ldots, D$. Define $\Psi_p = \{x_1, x_2, \ldots, x_K\}_{p_i}$ to be an attribute vector of $p_{di}$, then the set of products in the database is $P = \{p_{di} \mid d = 1, 2, \ldots, D\}$. In addition, all products in the database are further classified into mutually exclusive product groups as $P' = \{p_{d'i}(a_k) \mid d' = 1, 2, \ldots, D\}$, $i = 1, 2, \ldots, I$, each with $|P'| = D'$, and thus $\sum_{i=1}^I P = P$ and $\sum_{i=1}^I D' = D$. Then, the overall weight of a product group is specified by a vector $\Psi_p$, where $\Psi_p = \sum_{p_{di} \in P'}\Psi_{p_{d'i}}/D'$. The selling price of each product is defined by $s = [s_{11}, s_{12}, \ldots, s_{1d'}, \ldots, s_{1d'}, \ldots, s_{d1}, s_{d2}, \ldots, s_{d2}, \ldots, s_{d'}]$ and the possible profit is defined by $c = [c_{11}, c_{12}, \ldots, c_{1d'}, \ldots, c_{1d'}, \ldots, c_{d1}, c_{d2}, \ldots, c_{d2}, \ldots, c_{d'}]$, where $s_{di}$ and $c_{di}$ represent the corresponding price and profit of $p_{di}$, $i = 1, 2, \ldots, I, d' \in \{1, 2, \ldots, D'\}$.

3.1.1.2. Customers’ profiles. Denote a customer as $u_i$ with $f \in N$. Let $U = \{u_f(\theta) \mid f \in N\}$ be a set of the customers labeled by their demographic features $\theta \in \{\theta_1, \theta_2, \ldots, \theta_C\}$. To facilitate analysis, the customers are further classified into mutually exclusive customer groups as $U' = \{u_f(\theta) \mid f \in 1, 2, \ldots, F\}$, $j = 1, 2, \ldots, J$, and thus $\bigcup_{j=1}^J U' = U$. Furthermore, Define $B = [b_1, b_2, \ldots, b_j]^T, b_j \in [0,1]$ to be the average satisfactory level of customer group $U'$; and $B' = [B', B', \ldots, B']^T, B'$ to be the average budget of the customer group $U'$.

3.1.1.3. Relation matrix. Let $w_i^j$ denote the relative importance (weight) between product group P and customer group $U_i$, $i = 1, 2, \ldots, I, j = 1, 2, \ldots, J$. Then the relation matrix in between product groups and customer groups is defined by $w_{i,j} = [w_i^j \mid i = 1, 2, \ldots, I, j = 1, 2, \ldots, J]$ and this can be obtained from historical data and updated constantly.

Finally, define the decision variable as $x_i = [x_{i1}, x_{i2}, \ldots, x_{id'}, \ldots, x_{i1}, x_{i2}, \ldots, x_{id'}, \ldots, x_{id'}, \ldots, x_{id'}]^T, j = 1, 2, \ldots, J$, where $x_{id'} = 1$ if the $d'$-th product in $P$ group is recommended to $U_i$; otherwise $x_{id'} = 0$ for all $i = 1, 2, \ldots, I, j = 1, 2, \ldots, J, d' \in \{1, 2, \ldots, D'\}$.

Fig. 2. Flowchart of a recommender system.
3.2. The analytical model

Before developing the recommending model, two principles are based:

1. Recommending strategies take account of both utilities of EC company and customers.
2. The least information is acquired to provide the most satisfactory recommendation.

Then, when modeling, two assumptions are made:

1. Customer groups or product groups are mutually exclusive among the groups – therefore, each customer or each product only belongs to one and only one group.
2. Each customer or product can be presented by the features specified by the corresponding group.

For modeling, any product which does not satisfy the customer’s satisfactory level will be screened out before implement our model—therefore, the product list in our model are all feasible in this sense. In addition, because searching for a product is based on the weight \( w_i \) between customer group \( U_i \) and product group \( P \), in order to present individual features of a product in the groups, a parameter \( \beta_{i,j} \) is introduced to represent the degree of the product \( p_i \) belonging to its product group \( P_j \), as:

\[
\beta_{i,j} = 1 - \left( \left\| \frac{\psi_{p_i,j} - \psi_{p_i}}{T} \right\|^2 / K \right)^{1/2}.
\]  

(1)

where \( I \) is the levels of the product attribute \( x_k \) and \( K \) is the number of the product attributes. Eq. (1) says that the smaller is the difference between product and the group features, the larger the weight is in this group. Thus, we would present the relative importance between each product to the customer group \( U_i \) by:

\[
a^i = (\beta_{i,1} w_1, \beta_{i,2} w_2, \ldots, \beta_{i,J} w_J) = (\beta_{i,1} w_i, \beta_{i,2} w_i, \ldots, \beta_{i,J} w_i) \quad j = 1, 2, \ldots, J, \quad i = 1, 2, \ldots, I.
\]

(2)

Therefore, by taking account of customers’ preference and budget, a basic model for the on-line recommending module is proposed as follows:

(Basic model)

Maximize \( \sum_{j=1}^{J} c_j x_j^i \)

(2.1)

s.t. \( a^i x_j^i \geq b^i, \quad j = 1, 2, \ldots, J \)

(2.2)

\( s^i x^i \leq b^i, \quad j = 1, 2, \ldots, J \)

(2.3)

\[ \sum_{d=1}^{J} \sum_{d=1}^{D} x_{d,j}^i \geq 1, \quad j = 1, 2, \ldots, J. \]

(2.4)

\[ x_{d,j}^i \in [0, 1], \quad i = 1, 2, \ldots, I, \quad j = 1, 2, \ldots, J. \]

(2.5)

The size of the model is \( 3J \times D \), where \( J \) is the total number of customers considered in recommendation and \( D \) is the total number of products. This model describes that for an on-line recommending module, it is desired to maximize the profits of an EC company (2.1) when the products recommended to the customers satisfy their levels of satisfaction as shown in constraint (2.2); and the total price required to be spent on the products should not exceed the budget of the customer as shown in constraint (2.3). In addition, constraint (2.4) provides a tool for strategic uses by recommending different number of products of which at least one product will be recommended to a customer at each time.

4. Properties of the model

First, let us discuss the undesirable products which might be provided from the Basic Model. To avoid such situations, two strategies regarding market promotion are further developed. Additionally, the properties of optimal solutions under two strategies will also be discussed in this section.

4.1. Identification of undesirable products

One merit of utilizing analytical model is that from the infeasibility of the model, one is able to identify the undesirable outcome. That is, from the constraints (2.2) and (2.3) of the basic model, we may observe that there are two situations which will derive infeasible product:

1. The product \( p_i \) can not reach the customer’s satisfactory level: This can be identified by \( \beta_{i,j} w_j < b^i \) for customer group \( U_i \).
2. Individual price is higher than the budget: That is, \( s^i > b^i \) for customer group \( U_i \).

To avoid recommending such products to the customers, a subset of the database should be extracted by a pre-process so that the infeasible products will not be considered. This process, in turn, reduces the size of the data set. Denote the reduced decision vector and corresponding coefficients by \( \alpha'^i \), \( s'^i \). The Basic Model can be rewritten into Model (3) as below which will always provide desirable products:

Maximize \( \sum_{d=1}^{J} c^d x^d \)

s.t. \( a'^i x'^i \geq b^i, \quad j = 1, 2, \ldots, J \)

(2.6)

\( s'^i x'^i \leq b^i, \quad j = 1, 2, \ldots, J \)

(2.7)

\[ \sum_{d=1}^{J} \sum_{d=1}^{D} x_{d,j}^i \geq 1, \quad j = 1, 2, \ldots, J. \]

(2.8)

\[ x_{d,j}^i \in [0, 1], \quad i = 1, 2, \ldots, I, \quad j = 1, 2, \ldots, J. \]

(2.9)

where in Model (3), \( x^i = (x_j^1, x_j^2, \ldots, x_j^D) \) for customer group \( U_i \), and \( \alpha'^i, s'^i \) are the corresponding coefficients. Further investigating Model (3), one can realize that being desirable, which means that all products being considered have satisfied the customer’s preference level. Therefore, the constraint \( a'^i x'^i \geq b^i \) is redundant. By deleting these constraints from the model, four consequences are resultant:

1. If only one product is considered to be provided, then we can find the best choice simply by ranking the product’s coefficient \( c_{d,j} \) in the objective function, otherwise, we should recommend based on this model.
2. All the variables (products) left in Model (3) satisfy the similarity measurements i.e. \( \beta_{i,j} w_j > b^i \).
3. All products are treated as the same importance because the similarity measurements between products and customers are no longer meaningful.
4. The size of model is reduced from \( 3J \times D \) to \( 3J \times (\sum_{d=1}^{D} D_j) \).

4.2. Development of the market strategies

We now are at the position to utilize the reduced model to develop market strategies for an EC company, and three strategies are proposed and discussed in this section.
4.2.1. Maximal profit strategy

When the recommending processes takes solely from the viewpoint of the supplier, the goal will be to maximize the profits of the selling goods under a set of products that satisfy the customers’ preferences and budgets. When this is intended, the reduced model i.e. Model (3) will immediately reflect such strategy as shown in Model (4) below:

Maximize \( \sum_{j=1}^{l} c_j x_j \)

s.t. \( s'x_j \leq B_j, \quad j = 1, 2, \ldots, J \)

\( \sum_{j=1}^{l} \sum_{d=1}^{D} x_{jd} \geq 1, \quad j = 1, 2, \ldots, J \)

\( x_{jd} \in \{0, 1\}, \quad i = 1, 2, \ldots, l, \quad j = 1, 2, \ldots, J \) \quad (4)

The size of model is reduced to \( 2^J \times \left( \sum_{d=1}^{n} D_d \right) \). As what we have mentioned, the products derived from Model (4) of the Maximal Profit Strategy Model, would be all feasible and equally important to the customers. Therefore, based on the objective a supply company pursues, the recommended products are those which can provide the highest profit to the supplier when customers’ budgets are limited.

4.2.2. Win-win strategy

Although Maximal Profit Strategy will bring about the highest income to the suppliers, from management viewpoint, it may not retain customers. Therefore, an alternative strategy is considered and the proposed model provides such tool of analysis as follows.

By taking both suppliers and customers’ preferences into account, Model (5) below shows a Win–Win Strategy realization of which the first objective function maximizes the supplier’s profit as before; whereas the second objective function represents the maximization of the customers satisfaction.

Maximize \( \sum_{j=1}^{l} c_j x_j \)

Maximize \( \sum_{j=1}^{l} a_j x_j \)

s.t. \( s'x_j \leq B_j, \quad j = 1, 2, \ldots, J \)

\( \sum_{j=1}^{l} \sum_{d=1}^{D} x_{jd} \geq 1, \quad j = 1, 2, \ldots, J \)

\( x_{jd} \in \{0, 1\}, \quad i = 1, 2, \ldots, l, \quad j = 1, 2, \ldots, J \) \quad (5)

Model (5) is a Bi-criteria Programming model. By introducing the parameter \( \beta, \beta \in [0, 1] \), it can be transformed into a single objective programming model. As a result, we have a modified model as (6), which is called Win–Win Strategy Model. Note that in Model (6), \( c' \) is further normalized from \( c' \) into \( [0, 1] \) in order to reach the same scale with \( a' \).

Maximize \( \beta \sum_{j=1}^{l} c_j x_j + (1 - \beta) \sum_{j=1}^{l} a_j x_j \)

s.t. \( s'x_j \leq B_j, \quad j = 1, 2, \ldots, J \)

\( \sum_{j=1}^{l} \sum_{d=1}^{D} x_{jd} \geq 1, \quad j = 1, 2, \ldots, J \)

\( x_{jd} \in \{0, 1\}, \quad i = 1, 2, \ldots, l, \quad j = 1, 2, \ldots, J \) \quad (6)

Apparently when \( \beta = 1 \), Model (6) becomes Model (4). Therefore, Model (6) which is the same size with Model (4) is used in the module for supporting different strategies in management.

Besides, we would like to realize that when these two strategies result in the same recommendation list. This information is valuable because it provides the products which are not only maximizing the supplier’s profits but also fulfilling both parties’ desires simultaneously. In this paper, we call the above issue the Global Optimal Strategy which means to find a solution that is both optimal in the Maximal Profit Strategy Model and Win–Win Strategy Model. It is realized that this is required to find a threshold of \( \beta \) such that when we have an optimal solution for the Maximal Profit Strategy (\( \beta = 1 \)), this solution would also be optimal for the second objective with \( \beta \) decreased from 1 to the threshold. In this sense, the Global Optimal Strategy could be also regarded as a refined Win–Win Strategy. Because this issue is related to the optimal solution of the model, before we investigate this Global Optimal Strategy, let’s discuss the properties of the model for solution purposes.

Observe that both Maximal Profit Strategy Model and Win–Win Strategy Model can be basically regarded as one of the resource planning problems called Knapsack Problems (KP). In the following section, we shall briefly review KP. Then, from its properties, we can find the solutions for individual models of different strategies, and derive the rules of finding the threshold of \( \beta \) for the Global Optimal Strategy.

4.3. Knapsack-based solutions and the derived rules

A KP (Martello & Toth, 1990) is a problem saying that by giving a set of \( n \) items and a knapsack, with \( p_i \) the profit of item \( i \); \( w_j \) the weight of item \( j \); and \( c \), the capacity of the knapsack, how to select a subset of the items to be placed in the limited knapsack so that the profit is maximized. This problem can be formulated into the following model denoted as KP1 (7) and it can be noted that KP1 basically represents the Maximal Profit Strategy as our Model (4):

(KP1)

Maximize \( z = \sum_{j=1}^{n} p_j x_j \)

s.t. \( \sum_{j=1}^{n} w_j x_j \leq c \),

\( x_j = 0 \) or 1, \( j \in N = \{1, 2, \ldots, n\} \),

where \( x_j = \begin{cases} 1 & \text{if item } j \text{ is selected;} \\ 0 & \text{otherwise.} \end{cases} \) \quad (7)

Based on the basic assumptions of KP, we have \( \sum_{j=1}^{n} w_j > c \), and this assumption would leads to the fact that at least one item is chosen, which means the 2nd constraint of Model (4) and Model (6) i.e. \( \sum_{j=1}^{n} \sum_{d=1}^{D} x_{jd} \geq 1, j = 1, 2, \ldots, J \) would always hold in the KP.

Now define \( N^+ = \{j \in N: x_j = 1\} \) and \( N^- = \{j \in N: x_j = 0\} \), then \( N^+ \cap N^- \) is the optimal solution index set, and \( N^+ \cap N^- = \emptyset \). Then, let us consider the problem with the same solution set to the previous one, which is denoted by KP2 of Model (8). KP2 is the model that imposes a new parameter \( q_j \) into KP1, and is also the model used in corresponding to the Win–Win Strategy as in Model (6). Therefore, for these two KP model, \( p_i \) and \( q_j \) are used as the comparison basis to the profit(c_w) and similarity measurement(\( d_{ij}, w_i \)) respectively.

(KP2)

Maximize \( z = \sum_{j=1}^{n} (p_j + q_j) x_j \)

s.t. \( \sum_{j=1}^{n} w_j x_j \leq c \),

\( x_j = 0 \) or 1, \( j \in N = \{1, 2, \ldots, n\} \). \quad (8)
This is called a Linear Bi-criteria (LBC) knapsack problem (Geoffrion & Nauss, 1979). Let us use a vector form to describe the knapsack problem and the LBC model. Consider the vectorized form of a 0–1 knapsack problem as follows:

Maximize \( z = \mathbf{p}^T \mathbf{x} \)

subject to \( \mathbf{w}^T \mathbf{x} \leq c \),

\( x_j = 0 \text{ or } 1 \quad j \in \{1, 2, \ldots, n\} \).

(9)

In the LBC model, the objective coefficient \( \mathbf{p} \) is the composite of the weighted sum of two vectors as:

\( \mathbf{p} = \beta \mathbf{p}_1 + (1 - \beta) \mathbf{p}_2 \), \( \beta \in [0, 1] \)

(10)

Then \( z^{*} \)-value of a parametric solution is a piecewise convex linear function to \( \beta \) in the LBC model (Geoffrion & Nauss, 1979). In 1996, a network based approach for the construction of the parametric solution in a LBC model is proposed by Eben-Chaim, which is called the LBC-KP algorithm (Eben-Chaim, 1996). This algorithm solves the LBC problem iteratively with the specific \( \beta \) values by a network representation approach, and thus a parametric behavior could be constructed via the \( \beta \) values. The approach to determine the specific \( \beta \) values in each iteration is proposed by Eiser & Severance (1976), by finding the intersection points of each pair of extreme solutions corresponding to \( \beta_0 \) and \( \beta_1 \) respectively, \( 0 < \beta_0 < \beta_1 < 1 \). Since the LBC-KP algorithm is an efficient method to solve the LBC knapsack problem, it would be adopted in this paper for further analysis of \( \beta \) properties, which would be revealed in the following sections.

Recall that our goal is to know the issue that when the two strategies result in the same recommendation, the Global Optimal Strategy. This can be realized by determining that under what conditions would make KP1 and KP2 lead to the same optimal solution. In other words, with the given optimal solution from KP1, after imposing new parameter \( q_j \) into KP1 and turning out to be KP2, the optimal solution of KP1 remains the same as KP2. The issue described above is typically solved by a parametric analysis technique to find a parametric solution of KP2. Therefore, the LBC-KP algorithm is applied to construct a parametric solution via the domain of \( \beta \) and the threshold of \( \beta \) that ensures the Global Optimal Strategy could be determined. However, for on-line applications, the threshold of \( \beta \) may not be determined in time since the parametric analysis couldn’t be finished for the large scale of data. To overcome this insufficiency and serve an efficient on-line strategy, based on the properties of the model, we derive the following rule to find the threshold of \( \beta \), which is treated as the supplemental tool to ensure Global Optimal Strategy.

**Lemma.** If \( N^* = N^* \cap N^* \) is the optimal solution index set to KP1 with \( N^* = \{ j \in N: x_j = 1 \} \), \( N^* = \{ j \in N: x_j = 0 \} \) and \( N^* = \emptyset \). Then, if \( \text{Min}_{j \in N^*} \{ p_j + q_j \} > \text{Max}_{j \in N^*} \{ p_j + q_j \} \), the optimal solution remains in KP2, but not vice versa.

**Proof.** (\( \Rightarrow \)): Let \( k \) be the indexes such that \( p_k + q_k = \{ \text{Min} \{ p_j + q_j \} : j \in N^* \} \) and \( p_l + q_l = \{ \text{Max} \{ p_j + q_j \} : j \in N^* \} \), which is associated with KP1 and KP2. Then \( \text{Min}_{j \in N^*} \{ p_j + q_j \} > \text{Max}_{j \in N^*} \{ p_j + q_j \} \) implies \( p_k + q_k > p_l + q_l \). Let \( k \in N^* \), \( l \in N^* \), \( |k| = |l| = r > 0 \). The current optimal solution would remain if \( \sum_{j \in N^*} \{ p_j + q_j \} > \sum_{j \in N^*} \{ p_j + q_j \} + \sum_{j \in N^*} \{ p_j + q_j \} - \sum_{j \in N^*} \{ p_j + q_j \} \), so \( \sum_{j \in N^*} \{ p_j + q_j \} > \sum_{j \in N^*} \{ p_j + q_j \} + \sum_{j \in N^*} \{ p_j + q_j \} > \sum_{j \in N^*} \{ p_j + q_j \} \). Then \( \sum_{j \in N^*} \{ p_j + q_j \} > r \{ p_k + q_k \} > r \{ p_l + q_l \} > \sum_{j \in N^*} \{ p_j + q_j \} \), which induces the current optimal solution remains.

**Proof.** (\( \Leftarrow \)): We prove it by giving a counter example. As noted above, the current optimal solution would not change when \( \sum_{j \in N^*} \{ p_j + q_j \} > \sum_{j \in N^*} \{ p_j + q_j \} \). It holds that \( \sum_{j \in N^*} \{ p_j + q_j \} > r \{ p_k + q_k \} \) and \( r \{ p_k + q_k \} > \sum_{j \in N^*} \{ p_j + q_j \} \), and we can always find a example that \( r \{ p_k + q_k \} \leq r \{ p_l + q_l \} \). For instance, let \( V = \{ v_1, v_2, \ldots, v_m \} = \{ 1, 2, \ldots, m, v_1 < v_2 < \ldots < v_m \} \). Divide \( V \) into two subsets, \( A = \{ v_1, v_2, v_3, \ldots, v_m \} \), \( B = \{ v_2, v_3, \ldots, v_{m-1} \} \). Suppose \( v_1 + v_m + \ldots + v_n > v_2 + v_3 + \ldots + v_{m-1} \), we can still have that \( r \{ v_1 < v_2 < \ldots < v_n \}, r \in N \), which is the counter example. □

In summary, Rule 1 could be derived from the lemma and induced to find the threshold of \( \beta \) that ensures the Global Optimal Strategy as:

**Rule 1:** The optimal solution remains if \( \text{Min}_{j \in N^*} \{ \beta \{ p_j + q_j \} \} \geq \text{Max}_{j \in N^*} \{ \beta \{ p_j + q_j \} \} \), but not vice versa.

Rule 1 can be one way to check if the optimal solution remains, and illustrates the situation that when we have an optimal recommendation list, namely, List M, for Maximial Profit Strategy Model, if in List M, the minimal product's weighted value composed of its profit and similarity measurement is larger than the maximal weighted value of the product which is not in List M, then two models would yield the same recommendation. In this case, it implies that the products in List M are best choices regardless of the strategies adopted, and also these products are valuable for both supplier and customers.

5. Implementation of the on-line recommending module

Based on the properties of the proposed model and its availability, in this section, we shall introduce the on-line recommending procedure dividing into two parts of preliminary steps and main steps.

In the preliminary Steps, when a customer enters the recommender system, his/her demographic profiles (e.g. Age, Gender) are required to retrieve the customer's profiles (similarity measurements to products) from the off-line database. Then, the customer, who is noted as a member of \( U \), is asked to specify a satisfactory level \( d' \), and we initialize the model by setting \( b' \), the revision of \( d' \), which is used for rationalization of \( d' \) to fit the scale of the similarity measurements, and this is shown in (11.1)–(11.3) below:

\[ b' = (d') \times (\text{UBS} - \text{LBS}) + \text{LBS}. \]

(11.1)

\[ \text{UBS} = \text{Upper Bound of Similarity} \]

(11.2)

\[ \text{LBS} = \text{Lower Bound of Similarity} \]

(11.3)

When \( b' \) is determined, the Basic Model i.e. Model (2) could be constructed. Then based on two cases discussed in Section 4.1, undesirable products \( \mathbf{x}' \) can be identified by \( c_{d'w_{d'}} < b' \) and \( s_{d'} > b' \) for all \( \mathbf{d}' \) and thus can be eliminated from the possible recommendation list. If the extreme case that all \( s_{d'} \) are eliminated occurs, this shows no suitable products to meet the customer's requirement and we should ask the customer to lower the satisfaction level or increase the budget. Otherwise, denote the new decision variables to be \( \mathbf{x}'' \) and the corresponding coefficients to be \( \mathbf{a}'' \) and \( s'' \). Up to this stage, it can be referred to what we have proposed in Model (3), and the Preliminary Steps are finished. Then the Main Steps begin with the initialization of Model (6). If the Maximial Profit Strategy, Model (4) which is from Model (6) by defining \( \beta = 1 \) is implemented. If the Win–Win Strategy is adopted, giving a value of \( \beta \) between 0 and 1 (i.e. \( \beta \in (0,1) \)) with the smaller the value of \( \beta \) the higher the weight on the customers, then Model (6) is implemented. After running the model and a recommendation list is displayed with respective to the adopted strategy.
Since the recommender system is developed mainly for an EC company, additional information of the $\beta$ range to win the Global Optimal Strategy as a refined Win–Win Strategy would be beneficial and useful if the company requests for marketing purposes. Then, Rule 1 proposed in Section 4.3 as well as the LBC-KP algorithm will be implemented to find the threshold of $\beta$ and end the recommendation. In the follows, this procedure is summarized.

Preliminary Steps:

Step 1. Identify the customer’s demographic data and retrieve the customer’s profiles from the off-line database.

Step 2. The customer is asked to specify a budget $B^i$ and a satisfactory level $\delta^i$. Then the Basic Model is initialized by setting $b^i = (\delta^i) \times (\text{UBS} - \text{LBS}) + \text{LBS}$.

Step 3. Check the infeasibility condition and eliminate $x^i_{1d}$ if $\delta_{ij} w^j < b^i$ and $s_{ij} > B^i$ for all $i,d$.

Step 4. If all $x^i_{1d}$ are eliminated, ask the customer to lower $\delta^i$ or increase $B^i$. Otherwise, denote the new decision variables to be $x^i$ and the corresponding coefficients to be $a^i$ and $s$ as Model (3).

Main Steps:

Step 5. Set up Model (6). If Maximal Profit Strategy is adopted, set $\beta = 1$ and implement the model. Go to Step 6; if Win–Win Strategy is adopted, set a value of $\beta$ in $(0, 1)$ and implement the model. Go to Step 6;

Step 6. Display a recommendation list.

Step 7. If the Global Optimal Strategy is desired, find the threshold of $\beta$ and go to Step 8; otherwise end up the recommendation.

Step 8. Display the result and stop.

5.1. An illustrative example

In this section, we would present an illustrative example corresponding to the steps proposed in the Section 5. The example is meant to describe our proposed on-line recommending module with step-by-step illustration.

Giving six product groups, each contains two products. Based on an off-line database, information of similarity measurement $\delta_{ij} w^j$, relative weight $w^j$ are analyzed a priori. This example demonstrates how to select the optimal products for the customer according to his/her given conditions on the budgets and satisfaction levels with different market strategies:

Preliminary steps

Step 1

1. Define the customers’ attributes as $\omega_1$ (Gender = 1, 2) and $\omega_2$ (Age groups = 1, 2, ..., 6). Denote the entering customer as $u_1$ and customer is labeled as a member of $U^i$, where $U^i = \{u_{i}(1, 1) | f = 1, 2, ..., F^i\}$.

2. Retrieve the customer’s profiles (similarity measurements to products) and have the following information with respect to the product groups as shown in Table 2.

Step 2

The customer denoted as $U^i$ specifies a budget $B^i$ to be 1500 and his satisfactory level $\delta^i = 0.7$. Then the model is initialized by setting $b^i$ as below:

$$b^i = (\delta^i) \times (\text{UBS} - \text{LBS}) + \text{LBS}$$

$$= (0.7) \times (0.92 - 0.29) + 0.29 = 0.73,$$

where $\text{UBS} = \max \{\delta_{ij} w^j\} = 0.92$, $\text{LBS} = \min \{\delta_{ij} w^j\} = 0.29$.

Then the basic model is constructed as below:

$$\begin{align*}
\max 134x_{11}^i + 117x_{12}^i + 132x_{13}^i + 129x_{14}^i + 139.5x_{15}^i + 145x_{16}^i \\
+ 190x_{21}^i + 200x_{22}^i + 250x_{23}^i + 180x_{24}^i + 155x_{25}^i + 150x_{26}^i \\
s.t. \quad 0.70x_{11}^i + 0.92x_{12}^i + 0.80x_{13}^i + 0.71x_{14}^i + 0.73x_{15}^i + 0.66x_{16}^i \\
+ 0.39x_{21}^i + 0.45x_{22}^i + 0.31x_{23}^i + 0.29x_{24}^i + 0.69x_{25}^i + 0.80x_{26}^i \geq 0.73 \\
599x_{11}^i + 450x_{12}^i + 400x_{13}^i + 390x_{14}^i + 850x_{15}^i + 790x_{16}^i \\
+ 499x_{21}^i + 520x_{22}^i + 900x_{23}^i + 830x_{24}^i + 700x_{25}^i + 750x_{26}^i \leq 1500.
\end{align*}$$

$$\sum_{j=1}^{6} \sum_{d=1}^{3} x_{1d}^i \geq 1, x_{1d}^i \in \{0, 1\}, i = 1, 2, \ldots, 6.$$
Table 4
The objective values of all feasible solutions for two models

<table>
<thead>
<tr>
<th>Feasible solutions</th>
<th>(x_1)</th>
<th>(x_2)</th>
<th>(x_3)</th>
<th>(x_4)</th>
<th>(x_{12} x_{23})</th>
<th>(x_{12} x_{34})</th>
<th>(x_{12} x_{43})</th>
<th>(x_{12} x_{23} x_{34})</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximal Profit Strategy</td>
<td>0.780</td>
<td>0.800</td>
<td>0.930</td>
<td>1.000</td>
<td>1.660</td>
<td>1.710</td>
<td>1.780</td>
<td>1.810</td>
</tr>
<tr>
<td>Win–Win Strategy</td>
<td>0.850</td>
<td>0.840</td>
<td>0.830</td>
<td>0.900</td>
<td>1.690</td>
<td>1.680</td>
<td>1.750</td>
<td>1.670</td>
</tr>
</tbody>
</table>

Max 0.78x1 + 0.88x2 + 0.93x3 + 1x4
s.t. 450x1 + 400x2 + 850x3 + 750x4 ≤ 1500
\[ \sum \sum x_{ij} \geq 1, x_{ij} \in \{0, 1\}, i = 1, 2, \ldots, 6. \]

Taking \(\beta = (0, 1)\), we have the Win–Win Strategy model as:

Max \(\beta \cdot (0.78x_1 + 0.88x_2 + 0.93x_3 + 1x_4) + (1 - \beta) \cdot (0.92x_4 + 0.80x_2 + 0.73x_3 + 0.80x_1)\)
s.t. 450x1 + 400x2 + 850x3 + 750x4 ≤ 1500
\[ \sum \sum x_{ij} \geq 1, x_{ij} \in \{0, 1\}, i = 1, 2, \ldots, 6. \]

Step 6
Recommendation results:
If Maximal Profit Strategy is adopted with \(\beta = 1\), the optimal recommendation result would be \(x_1^*\) and \(x_4^*\).
If Win–Win Strategy is adopted with \(\beta = 0.5\), the optimal recommendation result would be \(x_1^*\) and \(x_4^*\).

Step 7
When the Global Optimal Strategy is desired, apply the LBC-KP algorithm (or Rule 1) to find the threshold of \(\beta\) to be 0.5455.

Step 8
Output the range of \(\beta = [0.5455, 1]\) and end the recommendation procedure.

5.2. Practice of different strategies

The merit of having a mathematical model is that it can provide different strategic analysis with systematic manner. In this section, we shall utilize this example to show such ability and discuss the results in details.

5.2.1. Maximal profit vs. Win–Win Strategies

For the example in Section 5.1, taking \(\beta = 0.5\), we would have the revised coefficients for Win–Win Strategy Model in Table 3 and all feasible solutions and optimal solutions (marked in boldface) for the two models in Table 4. By comparison, the Maximal Profit Strategy recommends the products i.e. \((x_1^*, x_4^*)\) with higher total profits 1.880 than 1.78 of the Win–Win Strategy.

Win–Win Strategy recommends the products i.e. \((x_1^*, x_4^*)\) with higher weighted value, 1.75, i.e. profits and similarity measurements than 1.74 of the Maximal Profit Strategy.

From the results, it is easy to recognize that these two models can be adopted for respective marketing purposes. From the procedure above, we obtain two different solutions from two strategies, which are extracted and shown in Table 5, and for simplification, we denote \([x_1^*, x_2^*, x_3^*, x_4^*]\) to be \([x_1, x_2, x_3, x_4]\). The column of weighted values in Table 5 can be regarded as the fulfillment degree for supplier’s profits and consumer’s satisfaction levels. In Maximal Profit Strategy of only the supplier’s profit being taken into account when the customer’s satisfaction level has to be at least 70%, we obtain the optimal products \((x_2, x_4)\). However, if only one product is to be recommended, \(x_4\) will be presented first since its price is the highest among the feasible solutions. On the contrary, if we take the Win–Win Strategy with equal weights for both supplier and customer (\(\beta = 0.5\)), different choices of \((x_1, x_4)\) would be revealed. Therefore, we would present \((x_1, x_4)\) to the customer. Again, if only one product is to be recommended, \(x_4\) is still the one presented first since its weighted value is the highest among the feasible solutions.

This example demonstrates that our model is designed to allow the recommender system to adjust priority between the supplier and customers; as well as the number of recommended items. Therefore, for the company, they can apply different strategies more objectively and flexibly for market expansion.

5.2.2. Global optimal strategy

From Section 4, we have discussed the condition which will allow the company to cover both strategies with minimum effort. As
we have mentioned in Section 4, LBC-KP algorithm would be applied to find the threshold of $\beta$ in a Global Optimal Strategy by constructing the parametric solution via the domain of $\beta$, which is shown in Table 6 with the parameters of the feasible solutions in the objective function, and in Fig. 3 with the results of the parametric analysis. In the parametric analysis appeared in Fig. 3, we could know that the range of $\beta$ is [0.5455, 1] for a Global Optimal Strategy, and the threshold is defined by its lower bound as 0.5455. Besides, from the rules specified in Section 4.2, $N^+ = \{2, 4\}$, $N^- = \{1, 3\}$. From Rule 1, the optimal solution is said to be the same under two models if $\min_{b \in \{2, 4\}} \left( \beta c_b + (1 - \beta) a_b \right) \geq \max_{b \in \{1, 3\}} \left( \beta c_b + (1 - \beta) a_b \right)$. We can verify Rule 1 by its Contrapositive Law, i.e., $\neg q \rightarrow \neg p$. In Table 5, it can be seen that the optimal solution differs from each other. From the Contrapositive Law of Rule 1, the optimal solution is different under two models if $\min_{b \in \{2, 4\}} \left( \beta c_b + (1 - \beta) a_b \right) < \max_{b \in \{1, 3\}} \left( \beta c_b + (1 - \beta) a_b \right)$. Therefore, it holds in our example by $\min(0.84, 0.90) < \max(0.85, 0.83)$. This evidence also shows that we could apply Rule 1 to check the initial condition for global optimality.

6. Conclusion and further research

In this paper, we have proposed an on-line recommending module that aims to achieve the decision support in the recommender system. In particular, a comprehensive model for evaluating different marketing strategies is focused. Efficient solution procedure has been developed and derived from its basic knapsack properties and its Linear Bi-criteria forms. Application of the proposed model with its possible marketing strategies was demonstrated and illustrated with a numerical example. In summary, this study has achieved the following goals:

(1) Development of a comprehensive model. This is our main contribution to the on-line recommending process of a recommender system. An analytical model is proposed by including the basic decision factors such as profits of business, customers' satisfaction levels and budget, and the adjustable priority of two parties' rights. With the proposal of a mathematical model embedded in the on-line recommending module, quantitative measurement can be better controlled; and performance of a recommender system can be well evaluated. It is also easier for the further researches that adopt this model to develop a recommender system with some modifications.

(2) Provision of three market strategies for the recommender system. We propose an analytical model with the parameter $\beta$ used to be corresponding to three market strategies, Maximal Profit Strategy, Win–Win Strategy and Global Optimal Strategy, in its different ranges.

When the recommending processes takes solely from the viewpoint of the supplier, Maximal Profit Strategy will be provided by setting up $\beta$ to be 1. To avoid the loss of customers in the long-run consideration, the system developer could also adopt a Win–Win Strategy model by setting $\beta$ in (0, 1). Then, within the framework of Win–Win Strategy, the Global Optimal Strategy is provided to reflect a conflict-free situation between Maximal Profit Strategy and Win–Win Strategy, and the ranges of $\beta$ in Global Optimal Strategy could be the reference for the system developer to set up the $\beta$ values.

In the future, a complete system should be developed which incorporates an off-line database with an updating mechanism. In the mean time, relevant elements of EC would be taken into consideration to enhance marketing competence such as bundling or cross-selling and attracting new customers.

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References


